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1 February 2015

Abstract. I explain how the growth of knowledge depends upon World 3 even though it involves creative discovery. I show the falsity of the oft-repeated claim that the conclusion of a valid argument is contained in its premises and I explain how that claim is a consequence of early modern conceptions of logical analysis, the structure of propositions and concepts, and the nature of necessary truth. I argue that inference, and also logical analysis, is a matter of guessing and testing, that is, a matter of discovery that depends upon World 3. I explain in what sense it is true that, as Karl Popper says, 'we never know what we are talking about.'

Keywords: analysis; analytic; a priori; building block; circular; compositionality; concepts; conjecture; containment; context principle; creative; decomposition; discovery; inference; informative content; logical content; necessary truth; *petitio principii*; Karl Popper; propositions; test; valid; World 3.

1. Introduction

I discuss the relationships between World 3, analysis and discovery, paying particular attention to the nature of inference. In section 2, I give an outline of Karl Popper's conception of World 3 and its connection with creative discovery. In section 3, I examine, criticise and reject the common claim that every deductively valid argument is circular. Why that false claim had come to be accepted by leading logicians is explained, in section 4, in terms of the early-modern conception of logical analysis and its connection with inference. In section 5, I say why there is no such thing as analysis as traditionally conceived; rather, analysis is a matter of conjecture and refutation. In section 6, I say explain why there is no such thing as inference as traditionally conceived; rather, inference is a matter of conjecture and refutation. In section 7, I briefly explain Popper's notions of logical content and informative content before explaining, in section 8, why 'we never know what we are talking about.' In section 9, I conclude.

2. Popper's Four Worlds

Popper distinguished four worlds, or four different parts of objective reality. *World 0*, which he does not label, is the world of pure abstract entities which are eternal and not made by humans. However, following Bolzano (1837, sections 19, 25, 30), Popper grants World 0 items only a kind of 'subsistence' rather than real existence: he says that their existence is 'shadowy' or 'virtual' or 'trivial' (1968a, pp. 116; 1968b, pp. 154, 158-59, including n8; 1994, p. 40). *World 1* is the physical world. *World 2* is the psychological world of mental states and events. *World 3* is the world of *objectified* abstract entities, including theories, problems, problem-situations, arguments, states of a discussion, stories, myths, oral traditions, works of art and so forth. World 3 is created by humans when they express World 0 entities in a physically realised code. Thus, humans *make* World 3 by giving World 0 items World 1 expression, thereby bringing them down to earth. Unlike World 0, World 3 grows over time (1968a, pp. 106-7, 112, 115-17, 122, 147; 1968b, p. 159).

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Popper normally uses 'World 3' to refer to World 3, but he sometimes uses it to refer to World 0 (1968b, pp. 154-55). The explanation is perhaps as follows. The *manifest* part of World 3 contains those abstract entities which have so far been given physical expression. But there are World 0 abstract entities which have so far not been given physical expression but which are related to the manifest part of World 3. Those entities make up the *latent* part of World 3 (1968a, pp. 116-18; 1968b, pp. 155-56). For example, when we invented the natural numbers we also invented, without realising it, prime numbers, the distinction between odd and even numbers, and theorems and problems concerning those numbers (1968a, pp. 118, 138; 1968b, p. 160; 1994, lecture 2).

Objective knowledge does not consist of individuals' psychological states; it consists of articulated theories, problems and arguments which have been objectified in language, particularly in articles, books and so on. Objective knowledge is only a part of World 3. It comes into existence with the creation of objectified *arguments*, because until then propositions, though true or false, are not *evaluated rationally* for truth or falsity (Popper 1959, preface to 1934 edition; 1968a, pp. 118-22, 147-49; 1968b, pp. 160-61; 1994, pp. 10, 13, 95-100). When we objectify our theories, we distance ourselves from them, which enables us to examine and criticise them more dispassionately, so that we can learn from ourselves. Such objectified knowledge can also be accessed and criticised inter-subjectively so that we can learn from each other even across the generations. Our study of objectified theories leads to the discovery of implications and problems that were unknown to the people who formulated the theories. The problems we discover stimulate us to develop new theories to solve those problems, thereby spurring the growth of knowledge. Thus, new theories are an outgrowth of the problem-situation created by prior theories; and the new theories create a different problem-situation which stimulates further discovery.

For instance, Einstein created general relativity theory which superseded Newton's theory. But if Einstein had lived two hundred years earlier, could he have come up with general relativity theory to replace Newton's theory then? I think we might allow that it is *possible*, because almost anything is possible; but it is extremely unlikely because general relativity was a product of Einstein's thinking about the problems that had emerged in physics after two centuries of work developing theories, such as field theory and the wave theory of light, to solve outstanding problems of Newtonian physics. Newton expressed the point himself: 'If I have seen further it is by standing on the shoulders of Giants' (1676, p. 416).

This is a *historical* conception of human knowledge, because future growth depends upon past achievements in that new theories are inspired by the problems discovered by the criticism and testing of *prior* theories. However, it is not a *cumulative* conception, because later knowledge typically revises earlier knowledge. The *revisionary* character of the growth of knowledge implies that advances in our knowledge – at least, the most significant advances – are not inferences from what we previously knew; they are, rather, *discoveries* which are the result of imaginative and creative problem-solving. A new discovery is a leap of imagination; but it is a leap that is informed and facilitated by the previous discoveries of others, so it is not entirely random (Popper 1974b, pp. 1061-62). The current state of our knowledge – the existing problem-situation – does not determine what will come next, but it does make some types of contribution more probable than others (Popper 1973, p. 128).

Popper (1968b, pp. 170-76) offers the example of Galileo's (unsuccessful) mechanical theory of the tides. Galileo's problem was to find a way of showing the superiority of heliocentric over geocentric astronomy. His tentative solution to that problem was the conjecture that the heliocentric theory would provide an explanation of the motion of the

tides. Thus, he had a new problem: how to explain the motions of the tides within the heliocentric theory. Galileo's problem-situation included the following constraints: the laws of inertia and the conservation law for rotary motions, which he took to imply that the planets have circular orbits; the rejection of any influence of the moon on the tides, in part because it was associated with astrology (which Galileo rejected) and in part because interplanetary influences seemed to be excluded by his conservation principle for rotary motions. That led to Galileo's conjecture that the motions of the tides are caused by the combination of the earth's orbital and rotating motions. Galileo's solution was not logically implied by his problem situation; but his proposal of it was made more probable in that context.

As an aside, various philosophers, including Philip Catton (2004), Imre Lakatos (1978), Alan Musgrave (2011a, pp. 218-25), John Worrall (1995, pp. 90-102) and Elie Zahar (1973, pp. 237-72) have tried to show that new theories are *deduced* from background propositions which are accepted by the person who proposes the theory. However, the background propositions that they invoke are often highly speculative metaphysical theories, or heuristic principles that correspond to such, that is, propositions which are themselves imaginative guesses not derived from previous knowledge. Their attempts to spirit away creative invention seem only to shift the lump around the carpet (Miller 2012, p. 14, might be making this point).

3. Petitio Principii

Petitio Principii is the fallacy of arguing in a circle or begging the question or assuming what is to be proved. Here is a gross example:

> God exists
> *Therefore*, God exists.

Here is a somewhat less gross example:

> God created the world
> *Therefore*, God exists.

Both arguments are *valid*. In a valid argument, it is necessary that, if the premises are true, then the conclusion is true (Aristotle 2007, I, 1); or, to put the point differently, but equivalently, it is impossible for the premises to be true and the conclusion to be false. The problem with a *petitio* is not that it is *invalid*, but that it is, usually, useless in disputation, because someone who does not already accept the conclusion of a *petitio* will normally not accept the premises either, because it is plain that the premises assume the truth of the conclusion.

A surprising claim that is often made by logicians and others is that *every* valid argument is a *petitio*. John Stuart Mill says:

> It is universally allowed that a syllogism is vicious if there be anything more in the conclusion than was assumed in the premises. But this is, in fact, to say, that nothing ever was, or can be, proved by syllogism, which was not known, or assumed to be known, before (1843, book 2 chapter 3, section 1, p. 197).

Augustus De Morgan, the man who gave us De Morgan's Laws, with which every student of symbolic logic will be familiar, says:

> It has often been asserted that all syllogism is a begging of the question, or a *petitio principii* in the modern sense, an assumption of the conclusion. That all premises do, when the argument is objectively considered, contain their conclusion, is beyond a doubt (1847, p. 257).

Why is the counter-intuitive claim, that every valid argument is circular or question-begging, 'universally allowed' and, supposedly, 'beyond a doubt'? Mill considers the following familiar argument:

(A) All men are mortal
Socrates is a man
Therefore, Socrates is mortal.

He says that the premise, 'All men are mortal,' assumes the truth of the conclusion, 'Socrates is mortal.' Anyone who doubted or denied the conclusion, that Socrates is mortal, would also doubt or deny the premise, that all men are mortal (1843, book 2 chapter 3, section 1, pp. 197-98).

De Morgan points out that what Mill says is not quite right, because someone may be ignorant that Socrates is a man (1847, pp. 257-59). For example, suppose that, on a foggy day, Socrates is pointed out to me. I see a shape in the distance and I now know it to be Socrates but, for all I know, Socrates may be a man, a bear, a statue or something else. So, even if I know that all men are mortal, I cannot validly infer that Socrates is mortal until I know that Socrates is a man. However, even with its second premise, says De Morgan, the syllogism (A) is not a gross *petitio*, and thus not a fallacy, because its conclusion is not *stated* in its premises, but is only *implied* in them:

> Every collective set of premises contains all its valid conclusions; and we may fairly say that, speaking objectively of the premises, the assumption of them is the assumption of the conclusion; though, ideally speaking, the presence of the premises in the mind is not necessarily the presence of the conclusion (1847, p. 254).

Richard Whately spells out the point:

> the object of all reasoning…is merely to expand and unfold the assertions wrapt up, as it were, and implied in those with which we set out, and to bring a person to perceive and acknowledge the full force of that which he has admitted;—to contemplate it in various points of view;—to admit in one shape what he has already admitted in another (1853, p. 258).

That, says Whately, means that, while every valid deduction is circular, only those which are blatantly circular involve the fallacy of *petitio principii*:

> *petitio principii* takes place when a premiss, whether true or false, is either plainly equivalent to the conclusion, or depends on it for its own reception. It is to be observed, however, that in all correct reasoning the premises must, virtually, imply the conclusion; so that it is not possible to mark precisely the distinction between the Fallacy in question [*petitio principii*] and fair argument (1853, p. 184).

The claim that in every valid argument the conclusion is contained, at least implicitly, in the premises, seems plausible for many arguments. It seems so for the valid syllogism (A) and also for the following slightly more complicated valid argument:

(B) All wealthy people play chess
If John is a phenomenologist, then John is wealthy
John is a phenomenologist
Therefore, John plays chess.

However, the claim does not seem plausible for some other valid arguments. For example (see Frederick 2014), let us stand argument (B) on its head:

(C) It is not the case that John plays chess
Therefore, it is not the case that: John is a phenomenologist, and if John is a phenomenologist then John is wealthy, and all wealthy people play chess.

That argument is valid. But none of the following assertions, taken from the logicians cited above, seems plausible:

- its complex conclusion is contained, or wrapt up, in its simple premise;
- there is nothing more in the conclusion than is assumed in the premise;
- anyone who admits the premise thereby admits the conclusion.

Imagine that I introduce my friend John to a person who has never studied philosophy and that I say 'It is not the case that John plays chess.' That person may understand the proposition I assert, but could he, by studying that proposition, arrive at the conclusion 'it is not the case that: John is a phenomenologist, and if John is a phenomenologist then John is wealthy, and all wealthy people play chess.' Surely not! For one thing, the person concerned has no idea what a phenomenologist is; and no amount or quality of inspection of the simple premise, 'It is not the case that John plays chess,' will ever provide him with such an idea. No propositions about phenomenologists are, in any intuitive sense, contained in that premise: no one can discover any such propositions by examining or reflecting upon the proposition expressed in the premise.

John Watkins offers a recipe for constructing somewhat similar examples:

> Take some powerful new scientific theory which has recently led to a striking new prediction. Formulate all the premises used in the derivation of this prediction. Among these there will almost certainly be a truism known long before the new theory was invented. Call this *a*, call the theory together with all the other premises *b*, and call the prediction *c*. Then an impressive implication of the truism *a* is: if *b* then *c* (1973, pp. 8-9).

But if we go back to a time before the powerful new scientific theory had been invented, anyone announcing the deductively valid conclusion, if b then c, from the premise, *a*, would surely have been saying something that was not contained in the premise.

It might be objected that, still, the following assertion of De Morgan is true,

- the conclusion of (C) is *objectively* contained in its premise;

so the claim made be the nineteenth-century logicians should be understood as saying that the conclusion of a valid argument is objectively contained in its premises. Unfortunately, before that claim can be understood we need an account of the notion of 'objective containment.' It will plainly not help the esteemed logicians to stipulate that a proposition is objectively contained in another when the former is a logical consequence of the latter. That would render their claim empty: it would say only that the conclusion of a valid argument is a logical consequence of its premises. Any informative thesis about containment will have gone by the wayside.

4. Analysis and Analyticity

Why was the claim that every valid argument is circular, because its conclusion is implicit, if not explicit, in its premises, 'universally allowed' by nineteenth-century logicians? The answer, I think, lies in three interrelated components of the early-modern conception of logic, namely:

(i) a building-block picture of concepts and propositions;
(ii) a decompositional conception of analysis;
(iii) a connection between necessary truth and analysis.

I explain these in turn.

The early modern philosophers, Rene Descartes, Pierre Gassendi, Antoine Arnauld, John Locke and their successors had a building-block conception of concepts and propositions. A concept, which was taken to be an *idea*, a mental item, was either simple or complex. A complex idea was composed out of other ideas which were its components; and a proposition, also conceived as mental, was composed out of ideas. Here are some passages from the *Port-Royal Logic*, by Arnauld and Nicole, which was the most influential logic book of the time, and which remained influential for two hundred years:

> The comprehension of an idea is the constituent parts which make up the idea, none of which can be removed without destroying the idea. For example, the idea of a triangle is made up of the idea of having three sides, the idea of having three angles, and the idea of having angles whose sum is equal to two right-angles, and so on (1662, p. 51).
>
> …every proposition is necessarily composed of three elements – the subject-idea, the attribute and the joining of these two ideas (1662, p. 109).

Some later thinkers regarded concepts and propositions as World 3 items which are apprehended by the mind, but they retained the building-block conception. Bolzano says:

> That there are parts of ideas which are ideas themselves has always been accepted and so requires no further justification…It is self-evident that every part of an idea must be connected with every other part, if not directly, at least indirectly. For they are both parts of one and the same whole, namely the one idea in which they occur (1837, section 58).
>
> …every proposition is a composite and includes ideas as its parts. Scarcely anyone will dispute that. For even in the simplest proposition, even if its linguistic expression

> consists of a single word, we shall nevertheless on closer inspection become aware of many parts, which are nothing other than individual ideas (1837, section 123).
>
> The comment was made about ideas…that every one was capable of being a component of another. The same thing is true of propositions, that every one can be considered as a component of another proposition (1837, section 124).

Similar thoughts are found in the later logicians, Gottlob Frege (for example, 1914, p. 225) and Bertrand Russell (for instance, 1903, pp. 47-51).

The early moderns were familiar with analysis in the sense of physical dismantling, as in anatomy, in which the body is broken apart into its various components, and also from alchemy and early chemistry, in which the different constituents of a substance are separated. An analogous conception of analysis was familiar from geometry, where one tries to discover facts about a geometrical figure by bisecting it or splitting it into component figures (Koertge 1980, pp. 144-151). That is only *analogous*, because the geometrical figure is not literally broken up but is only imagined to be so. Similarly, in grammar, sentences and terms, the instances of which are spatial figures or temporal sounds, were, in imagination, split up into their component terms. Logic is concerned with concepts, propositions and arguments, which belong to World 0; but it deals with such entities only insofar as they have linguistic expression, and thus belong to World 3. Logicians had followed grammarians in mentally decomposing sentences and terms into component expressions and, given the building-block conception of concepts and propositions, it was natural for them to assume that propositions and complex concepts could be dismantled into component concepts. However, use of the term ‘analysis’ in relation to concepts and propositions is even more metaphorical than its use in geometry and grammar, because concepts and propositions are not spatial or temporal. That point may have been hidden to the extent that concepts were taken to be ideas in the mind and the latter were thought, especially by empiricist philosophers, to be, or to be like, visual images. In short, it was thought that propositions and concepts are items which are open to inspection by the mind and that, if we examined them intently enough, we would be able to see the component concepts which constituted them.

Thus, in his *Logic*, which was part of his influential *Syntagma*, Gassendi says (1658, pp. 368-69) that, if we want to prove that man is a substance, we may proceed by resolving ‘man’ into his genus ‘animal’ and his differentia ‘rational.’ We then analyse ‘animal’ into ‘living’ (genus) and ‘sentient’ (differentia), and we similarly resolve ‘living’ into ‘body’ and ‘animate.’ Since a body is anything endowed with mass, and anything endowed with mass is a substance, we understand that the genus of ‘body’ is ‘substance.’ That passage from Gassendi shows the close connection between the idea of analysis and that of inference, which we can exhibit by converting the passage into a sequence of valid inferences in each of which a proposition appears to be drawn from another by means of analysis of a component concept:

(1)	Every man is rational and an animal	[true by virtue of analysis of *man*]
(2)	Every man is living and sentient	[from (1) by virtue of analysis of *animal*]
(3)	Every man is animate and body	[from (2) by virtue of analysis of *living*]
(4)	Every man has mass	[from (3) by virtue of analysis of *body*]
(5)	Every man is a substance	[from (4) by virtue of analysis of *mass*].

Gottfried Leibniz (1679; 1686) and Immanuel Kant (1781/1787, Introduction, section IV) maintained that necessary truths are those that can be known by means of analysis of the concepts that they contain. There are two complications in their views that I will mention only to put on one side. First, Leibniz thought that *all* truths were knowable by analysis but that contingent truths required an infinite analysis which we were unable to carry out. Second, Kant thought that some necessary truths are knowable, not by analysis of the concepts they contain, but by analysis of the conditions of the possibility of experience. However, we can neglect those idiosyncrasies here. What became a standard doctrine in philosophy is that necessary truths are knowable by analysis of their content. When that is combined with the building-block conception of concepts and propositions, and with the decompositional conception of analysis, we end up with the false doctrine that the conclusion of a valid argument is contained in its premises, as I will now show.

Kant used the concepts of 'subject' and 'predicate' to explain the relevant notion of analysis. Thus, propositions like (1) are 'analytic' because the concept of being a man includes as parts the concepts of being rational and of being an animal. Such propositions are 'explicative.' They merely set forth more clearly what our concepts already contain, so they can be known by analysis. They are therefore necessary. In contrast, propositions like

All men are honest

are contingent because the concept of being honest is not a part of the concept of being a man. Such propositions are 'ampliative.' They go beyond analysis to say something new. We can know them only by appeal to experience. Leibniz's explanation (1765, p. 362) of necessary truths, which we can know by analysis, also talks of propositions in which the predicate is contained in the subject, such as

An equilateral rectangle is a rectangle

but in addition he includes conditional propositions in which the consequent proposition is contained in the antecedent proposition, such as:

> If a figure with no obtuse angle can be a regular triangle, then a figure with no obtuse angle can be regular.

Every argument corresponds to a conditional proposition which takes the conjunction of the argument's premises as antecedent and the argument's conclusion as consequent. So, to syllogism (A) there corresponds the following conditional:

(a) If all men are mortal and Socrates is a man, then Socrates is mortal.

If the argument (A) is valid, it is necessary that, if the conjunction of its premises is true, then its conclusion is true, which means that the argument's corresponding conditional, (a), is a necessary truth. Now, if all necessary truths are analytic, then the conditional corresponding to a valid argument is analytic, that is, on the Leibniz-Kant view, the proposition expressed in its consequent is contained in the proposition expressed in its antecedent. If that is so, then, on the building-block conception of propositions, and the taking-apart conception of analysis, we can discover the consequent by analysis of the antecedent. Therefore, on this view, the validity of an argument implies that its conclusion is contained in its premises and that we

can find that conclusion in the premises if we mentally examine the premises intently enough. As Kant's contemporary, Moses Mendelssohn put it:

> what else can the profoundest inferences do but analyze a concept and make distinct what was obscure? Such inferences cannot bring in what is not to be found in the concept, and it is easy to see that it is also not possible, by means of the principle of contradiction, to derive from the concept what is not to be found in it…The analysis of concepts is for the understanding nothing more than what the magnifying glass is for sight. It does not produce anything that was not to be found in the object. But it spreads out the parts of the object and makes it possible for our senses to distinguish much that they would otherwise not have noticed (1763, pp. 257-8, quoted in Beaney forthcoming).

The same thought seems to underlie the famous passage about deriving an 'ought' from an 'is' in Hume's *Treatise*: 'a reason should be given, for what seems altogether inconceivable, how this new relation ["ought" or "ought not"] can be a deduction from others ["is" or "is not"], which are entirely different from it' (1740, p. 469).

Mill (1843, book 1, chapter 6, sections 1-4, pp. 147-51) and De Morgan (1847, p. 331) explicitly endorsed the three traditional dogmas (i) - (iii) about concepts, analysis and necessary truth. I guess that nineteenth-century logicians generally accepted them and that that is the explanation for why they 'universally' held that all valid arguments are circular.

However, we have seen that there are many valid arguments the conclusions of which are not contained in their premises, and the corresponding conditionals of those arguments, though necessary, are not analytic in the sense that the consequent can be derived by disclosing it within the antecedent. It follows that the conjunction of the three traditional dogmas, (i), (ii), (iii), is false. As it happens, I think that each of (i), (ii) and (iii) is false, and I will briefly say why.

5. 'There Is No Such Thing As Analysis'

Obviously, I do not deny that there is such a thing as analysis, in the sense of dismantling, with regard to physical objects, as in anatomy, physics and chemistry, or, in an analogical sense, in geometry and grammar. However, there is no such thing as dismantling in the case of concepts or propositions because the building-block conception of such abstract entities is mistaken. The idea that propositions are built out of concepts, or out of any other elements, leads directly to the problem of the unity of the proposition. If a proposition can be analysed into a set of elements, then all you have is a collection of items, not a proposition which says something true or false. Russell (1903, pp. 49-50) struggled with that problem but despaired of a solution. Later he adopted Frege's solution (1892, pp. 54-55), which invokes the notion of the 'unsaturatedness' of concepts. However, Frege's solution is mystical and ad hoc, and it does not even solve the problem. It seems that the only way to solve the problem is to deny that propositions have components at all; though, of course, we must recognise that propositions are *related* to concepts. For example, the proposition that all men are mortal is true if and only if everything that instantiates the concept *man* also instantiates the concept *mortal*. That fact does not require the concepts to be *parts* of the proposition. I explain all this in a paper which is available online (Frederick MS).

The same applies to concepts. Concepts do not have other concepts as parts: they are not the sort of entity that can have parts. Since no concept is compounded of other concepts, there can be no analysis which is a dismantling of concepts. However, concepts do have logical relations of involvement and exclusion with other concepts. When theorists say, for

example, that the concept *human* contains the concept *animal*, they are saying something which is literally false; but there is a truth that lies behind their statement, namely, that the concept *human* logically involves the concept *animal*. That is to say that 'Socrates is a human' entails 'Socrates is an animal' and similarly for other pairs of such subject-predicate propositions. Propositions about concept involvement are propositions about entailment relations between propositions. Similarly, to say that the concept *human* excludes the concept *horse* is to say that 'Socrates is a human' is incompatible with 'Socrates is a horse' and similarly for other pairs of such subject-predicate propositions.

Once we shift our attention from focusing on a particular concept to investigating its logical relations to other concepts, there seems to be no limit to the scope of our enquiry, since we can try to discover what relations our familiar concepts have to newly formulated concepts. Indeed, the formulation of new concepts *is* a matter of clarifying their relations to existing concepts.

Philosophers who are proponents of 'conceptual analysis' have more or less come to recognise this. For some decades many of them, including A. J. Ayer (1946, pp. 31-33), Gilbert Ryle (1949, p. 9) and P. F. Strawson (1959, pp. 9-10; 1992, pp. 18-20), have preferred to describe their enterprise as something like conceptual geography which traces the relations between different concepts, rather than breaking down a particular concept into its components. We can see that something similar applies to geometry too. For while, as I said in section 4, geometrical analysis often involves mentally breaking down a figure into components, it also often involves relating the figure to other figures. That is evident, for example, in the passage in Plato's *Meno* (1956, pp. 130-37) in which Socrates supposedly shows that the slave-boy has geometrical knowledge that he has never been taught but which is in some manner in abeyance. For, to show that the diagonal of a square is the side of a square which has twice the area of the original square, Socrates does not only break the original square into two triangles, but also relates the original square to three other squares, and their components, which he draws. I said that Socrates only supposedly shows that the slave-boy had prior geometrical knowledge because Socrates is actually *teaching* the boy some geometry: he produces a demonstration and asks the boy questions the answers to which seem obvious given comparisons that Socrates has laid out.

But how does one go about discovering the logical relations between concepts? Is it a matter of mentally gazing at a collection of concepts in order to intuit their various relationships? Although some philosophers do write as if that is what is done (for example Huemer 2005, pp. 125-26)), I suggest that there is no such activity. What philosophers actually do when they engage in so-called conceptual or logical analysis is guessing and testing. They develop theories about the logical relations between concepts, that is, they make conjectures about entailment or incompatibility relations between propositions, and they then test those conjectures by looking for counter-examples; or they develop the conjectures by means of tentative derivations which they then test by looking for counter-examples. That, indeed, seems evident from the Socratic dialogues. For theorists who follow Frege and Russell, guessing and testing a conjecture about the logical relations of types of proposition typically involves guessing and testing how propositions of that type should be put into logical form (as with Russell's Theory of Descriptions). The same applies in geometry: in the *Meno*, Socrates made a guess about the diagonal of a square and then tested it by drawing the other three squares and their diagonals and counting the four half-squares.

So, there is no such thing as conceptual or logical analysis, if such analysis is supposed to be something different to guessing and testing. We may, however, describe conceptual or logical analysis as *a particular kind* of guessing and testing, namely, guessing

and testing a theory about logical relationships. Indeed, that seems to be exactly what Popper means when he speaks, as he so often does, of 'logical analysis' (1959, pp. 27-28, 38 n *1, 180-81, 183-84, 235, 316).

Having rejected the building-block conception of concepts and propositions, and the decompositional conception of their analysis, we must go on to reject the Leibniz-Kant explanation of necessary truth in terms of containment of one concept within another, or of one proposition within another. Indeed, once we replace talk of containment with talk of entailment relations between propositions, the Leibniz-Kant explanation of necessary truth begins to look circular, because it then explains necessary truth in terms of entailment; but entailment is explained in terms of necessary truth (see section 3, above).

However, it appears that the explanation of necessary truth in terms of entailment is not merely circular but *mistaken*, because there are some necessary truths which are not correlated with statements about entailment. For example, statements of laws of nature are necessarily true, if true, as Popper conceded (1959, appendix *x) in his debate with William Kneale (1949, chapter 2). Thus, suppose that Newton's Law of Gravity is a law of nature. Then, it is a necessary truth that, if Socrates has mass, then Socrates generates a gravitational force; but it is not the case that 'Socrates has mass' entails 'Socrates generates a gravitational force.' Newton's Law of Gravity was an empirical discovery not a logical one (see Kripke 1980 for extended discussion of this sort of issue). Thus, necessary truth of a conditional is only a necessary, not a sufficient, condition for the antecedent of the conditional to entail its consequent.

I suggest that we should take the notion of necessity as primitive or, at best, explain it in terms of the other modal notion of possibility. For example, we may say that a proposition is necessarily true if and only if it is not possible for it to be false. But we also need a way of distinguishing those necessary truths which ground entailments from those which do not. The difference seems to be that the former are those which are knowable a priori. However, the notion of a priori knowledge is a dubious one; so I suspect that we can provide only a tentative demarcation between the necessary truths which ground entailments and those which do not, and a demarcation which is fuzzy at that.

Given that contemporary analytic philosophers generally reject the de-compositional notion of conceptual or logical analysis in favour of a relational notion, it is surprising that the claim that the conclusion of a valid argument is contained in its premises is still commonly maintained, for example, by Michael Dummett (1981, p. 291; also 1973, p. 297), David Miller (1994, pp. 55-58), Alan Musgrave (2011b, p. 2), Alan Hausman et al. (2013, p. 5) and William Hughes and Jonathan Lavery (2004, p. 197). That appears to be a failure to 'put two and two together.'

6. 'There Is No Such Thing As Inference'

If there is no such thing as analysing concepts and propositions by breaking them down into their components, then there is no such thing as inference, *if* inference is supposed to be drawing from a simple or compound premise a proposition which is discovered by analysis to be contained within it. How do we arrive at a valid conclusion from a set of premises?

A view that has been popular amongst philosophers and psychologists in recent years, as well as historically, is that arriving at a conclusion from a set of premises is an automatic process, often conceived to be computational or calculative, involving the application of formal rules of inference. The process need not be conscious: it may proceed sub-consciously or even sub-personally. Variants of this view have been propounded by Aristotle (1955, VII, 3; 2004, 7), Thomas Hobbes (1651, pp. 81-82), Jerry Fodor and Zenon Pylyshyn (1988), Bill

Brewer (1995, pp. 241-42), Christine Korsgaard (1996, p. 320), Martin Davies (2000, pp. 96-103), John Broome (2002, p. 95; 2009, section 4), and many others.

There are four problems with that type of view. First, some deductively valid arguments are not *formally* valid, such as:

> The ball is red
> Therefore, the ball is coloured.

Such inferences cannot be reduced to the application of formal rules of inference, except in an ad hoc way, by supplying tailor-made 'meaning postulates.' Second, an automatic process is not a process of reasoning, because reasoning requires an agent who *appraises* arguments in view of his *aims*, principally problem-solving, and his *values*, normally involving preferences for truth and validity; none of which can be literally true of an automaton (Kant 1785, 1: p. 448; Popper 1982, pp. 81-85). Our cognitive processes do seem to *depend upon* automatic calculative processes, for example, in depth perception; but those processes are not *reasoning* processes, despite what some psychologists say. Third, our awareness of our fallibility means that our reasoning cannot be automatic: it is open to a reasoner to doubt or deny any rule of inference, as the existence of a variety of deviant logics shows (Priest and Thomason 2007, pp. 96-98). So reasoning cannot be specified as a process that follows a specific set of rules of inference. Fourth, even if we know that a reasoner accepts a specific set of rules of inference, he may doubt or deny any instance of inference in accordance with those rules, as Lewis Carroll (1895) showed.

In contrast, I suggest that deduction is a process of guessing and testing. When we infer a conclusion from a set of premises, we do not 'draw the inference' or 'elicit the conclusion' from the premises; and nor do we go through an automatic process, whether or not rule-governed or calculative. Rather, we *guess* the conclusion and then, if we have time, we *test* that guess. There are various ways in which we can test such a guess. For example, we may put our premises and tentative conclusion into logical form and then use one of the algorithms available in propositional or predicate logic, such as truth tables or reduction to conjunctive normal form or semantic tableaux, to discover whether our tentative conclusion is a valid one. Alternatively, we may try to construct a derivation of our tentative conclusion from our premises using recognised rules of inference; or we may look for a counter-example, that is, a possible situation in which the premises would be true but the conclusion false. If our conjecture about validity survives testing, we have made a *tentative* discovery that the conclusion follows from the premises. The discovery is tentative because none of our tests is infallible. For example, contemporary propositional and predicate logic is the best system of formal logic we have ever had; but it can be improved and it may also be mistaken. For instance, the rule of existential generalisation is unsound when applied in natural languages because it fails to recognise that singular terms may appear as predicates (Frederick 2013); and the rule of universal specification is debatable because it makes it a theorem that there exists at least one object, but some theorists regard that proposition as contingent (they deny that there are any necessary existents).

Thus, inference is a matter of discovery, not a matter of analysis as traditionally conceived. A related point is made by Popper (1974c, pp. 1086-87; 1975, p. 13), Willard Quine (1951, p. 87) and others, and has been emphasised by Imre Lakatos (1976), namely, that coming up with a 'proof' is a creative process in which we guess a conclusion and guess possible derivations and then proceed to test them. The process can, to some extent, be simulated mechanically by computers (Gillies 1996, pp. 85-94); but such 'machine guessing'

(Miller 2012, pp. 15-17) is only a *simulation* of inference precisely because it is mechanical, whereas inference is a goal-directed activity sensitive to values, and thus teleological. Mechanical simulation is also currently *limited* because, while we can programme a machine to discover proofs within a particular formal logic, such as first-order predicate logic, it seems that a machine is not be able to challenge an existing proof by making some conceptual modification which may lead to a new system of logic. Perhaps in time such conceptual innovations could also be simulated by machines; but they would remain simulations insofar as they are mechanical rather than teleological. On the other hand, if we are eventually able to build a machine that thinks – genuinely thinks, rather than simulates thinking – then we will have created a person, something teleological rather than purely mechanical. (I am not here claiming that we know that we are not machines. My claim is that persons are teleological and that we *think* that we are persons. If, on the contrary, we eventually discover that we are in fact machines, we will thereby discover that we are not persons. See Frederick 2012.)

In this light, reconsider our earlier arguments. In argument (A) we have the premises:

All men are mortal
Socrates is a man.

We guess that 'Socrates is mortal' follows. We may then test that guess by formalising the argument in predicate logic, to get

$(x)\ (Fx \rightarrow Gx)$
Fa
Therefore, Ga

and then use a semantic tableau to check for validity. It might be said that there is no need to test argument (A) for validity because it is *obvious* that the conclusion follows. It is true that the attempt to test *everything* would lead to an infinite regress, because we would then have to test our tests and then tests the tests of our tests, and so on. However it is also true that we can test *anything*. Even checking whether it seems obvious that a conclusion follows is a test. It is not, though, a very effective test because many things that have seemed obvious have turned out to be false, such as the absoluteness of simultaneity, or Frege's fifth axiom for arithmetic which Russell showed to be self-contradictory. But all our tests are fallible.

With respect to argument (B), the conclusion, 'John plays chess' may also seem quite obviously to follow from the premises:

All wealthy people play chess
If John is a phenomenologist, then John is wealthy
John is a phenomenologist.

However, I say 'quite obviously' because the degree of obviousness is less than in argument (A) given that the premises of argument (B) are more numerous. Again, we could convert premises and conclusion into formulae of predicate logic and then test for validity using a semantic tableau. Alternatively, we may construct a derivation informally. Thus, from the third premise and the second premise we may infer (that is, guess) that John is wealthy; and then from that proposition and the first premise we may infer (that is, guess) the conclusion of the argument. We might test these guesses by checking how obvious they seem or by trying to construct counter-examples.

In the case of argument (C), as we noted, there seems to be nothing in the simple premise, 'It is not the case that John plays chess,' to suggest as a conclusion the complicated proposition:

(p) it is not the case that: John is a phenomenologist, and if John is a phenomenologist then John is wealthy, and all wealthy people play chess.

A person would have to go out of his way to think up (p) given only the simple premise, especially if he had not previously studied philosophy and had no idea what a phenomenologist is. However, once he has (p), he can test whether it follows from the simple premise, for example, by putting premise and conclusion into logical form and using a semantic tableau, or by constructing an informal derivation the first step of which could be to show that argument (C) is equivalent to argument (B) and then informally deriving the conclusion of (B) from its premises as before.

It may be wondered: why would anyone considering 'It is not the case that John plays chess' come up with the proposition (p) as a possible conclusion? Clearly, it is highly unlikely that such thoughts should come out of the blue. A person would have to have been thinking about the connections between phenomenologists and wealth and chess-playing before the thought of the premise of (C) would lead him to the thought that argument (C) is valid. In short, the inference would be likely to occur to someone only in some particular types of problem-situation.

Argument (C) is a simplification of an example that was provided by Popper (1974a, pp. 18-21; discussed in Frederick 2011), namely, that Newton's theory of gravity entails the negation of Einstein's theory of gravity. Popper says that this entailment could not have been known to Newton or to anyone else prior to the discovery of Einstein's theory. But, once Einstein had come up with the theory, its incompatibility with Newton's theory would have been clear to him (Einstein's theory says that there is no force of gravity, but Newton's theory says that there is such a force), and Einstein would have been very interested in that logical fact. As we saw earlier, it is what Popper calls the 'problem-situation' that stimulates the creation of hypotheses that would hardly have been thought of in an earlier, very different problem-situation, and which leads thinkers to conjecture that a new complex hypothesis, or its negation, is entailed by another. Thus, valid arguments are not the outcome of the analysis of premises but are rather discoveries which are facilitated or enabled by particular World 3 problem-situations.

We noted in section 2 that new theories are not inferred from prior knowledge, but are guessed in the light of prior knowledge. Yet I have proposed that when we infer a conclusion from a set of premises we are making a guess. Why does this not make Popper's conjecture-and-refutation view compatible with inferring theories from prior knowledge? The answer is that when we make an inference, we are making a guess about *what follows* from a set of premises, but when we propose a new scientific theory, we are making a guess about *what explains* some set of phenomena. We test a guess about validity by attempting to construct a derivation or by seeking a counter-example which shows that the conclusion may be false while the premises are true. We test a guess about explanation by comparing it with rival guesses and by seeking a counter-example which shows that the proposed explanation is inconsistent with a basic statement. Thus, the notion of *inductive inference* confuses guesses about validity with guesses about explanation. The confusion is presented starkly in the phrase 'inference to the best explanation:' a guess about explanation is not a guess about what follows from what and is thus not an inference.

7. Logical Content, Informative Content and Containment

Popper defines the *logical content* of a proposition as the class of all the non-tautological propositions which are derivable from it (1959, section 35; 1974a, p. 18). Two things should be noticed about that definition. First, the notion of content is explained in terms of the notion of derivability, or validity. In contrast, the traditional claim we have been criticising attempted, unsuccessfully, to explain validity in terms of content (the conclusion being part of the content of the premises). Thus, Popper takes an opposite approach: on his view, as on mine, it is logical relations between propositions that do the explanatory work. Second, Popper's definition makes it *false* that in every valid argument the conclusion is part of the logical content of the premises. For, every proposition entails an infinity of tautologies; but Popper excludes tautologies from the logical content of propositions. So, on Popper's definition of 'logical content,' the conclusion of the following valid argument is *not* part of the logical content of its premise:

> Socrates is a man
> *Therefore*, either 1 = 2 or it is not the case that 1 = 2.

In short, unlike the traditional notion of *containment* of concepts or propositions within others, Popper's notion of *logical content* is *not* intended to be part of an explanation of when, or why, one proposition entails another. Popper introduces his notion as part of the explanation of falsifiability, that is, *empirical content* (1959, especially sections 35-36). His interest is in explaining *testability*, not *validity*.

In his earlier work Popper identifies the *informative content* of a theory with its empirical content, that is, with the class of basic statements which are inconsistent with the theory (for example, 1959, appendix *ix). However, he later (1974a, p. 18) defines the informative content of a proposition as the set of propositions which are incompatible with it. Thus, general relativity is part of the informative content of Newton's theory and vice versa. A theory is more informative the more it excludes. We must therefore question Hume's 'is'-'ought' dichotomy. If we accept the principle that 'ought' implies 'can,' then we accept that 'cannot' excludes 'ought.' That is, propositions about what people *cannot* do have propositions about what those people *ought to do* as part of their informative content. But I mention that only in passing.

8. 'We Never Know What We Are Talking About'

We have seen that many of the logical consequences of a proposition cannot be known by us because they need to be discovered by future generations. However, our understanding of a proposition depends, at least in part, on our knowing what its logical consequences are: if a proposition has some logical consequences of which we are ignorant, to that extent we do not understand the proposition. It follows that we can never *fully* understand any proposition, which is a point Popper makes by saying that **'**we never know what we are talking about' (1974a, pp. 19-20). Similarly, understanding a concept involves knowing its logical relations to other concepts which, as we saw in section 5, means knowing the logical relations between propositions involving the concept and propositions involving other concepts. But since it is always the case that knowledge of the logical relations between a given proposition and some other propositions depends upon discoveries yet to be made, we can never *fully* understand any concept. Popper infers: 'There simply is no such thing as a precise concept' (1974a, pp. 21-22).

These comments bear upon a long-standing problem in the philosophy of language. One principle that is commonly accepted is that we understand the meaning of a sentence by understanding the meanings of its component words and their mode of combination. There must be something in this principle because understanding a language enables us to understand a potential infinity of sentences which we, being finite, can accomplish only because sentences are built up from a finite stock of components (words and grammatical constructions). This *compositionality principle* is found in Frege (1914, p. 225) and in many other theorists. In apparent conflict stands the *context principle*, also found in Frege (1880/81, p. 17; 1884, p. x), that only in the context of a sentence does a word have any meaning. That principle suggests that it is by understanding sentences, or the truth-conditions of sentences, that we come to understand the meanings of words. There is something in that view too. For, one cannot understand a word without understanding the logical role that it plays in sentences. For instance,

- a term, such as 'red,' connotes a property which may be truly or falsely attributed to things;
- a sentential connective, such as 'and' has the function of joining two sentences to make a third, the truth-conditions of which depend upon the truth-conditions of the sentences joined;
- a copula, such as 'is,' joins a singular term and another non-relational term to make a sentence which is true if and only if the thing denoted by the singular term exemplifies the property connoted by the other term.

Indeed, we may come to understand the meaning of a word by understanding some sentences in which it us used.

However, although there is something in each of the compositionality and context principles, strictly speaking both are false, because they miss the bigger picture. As we have seen, we can never fully understand a word or a sentence, because to do either we need knowledge about logical consequences which is simply not available to us. The fact that a language must be composed of a finite number of components, which may be combinable in a potential infinity of ways, implies only that we must understand *part of* the meaning of a word before we understand new sentences in which it is contained. The fact that words are made to be parts of sentences implies only that *part of* the meaning of a word that we must understand, in order to be able to understand sentences containing it, is its grammatical function or logical role; and when we come to understand a word by understanding some sentences in which it is used, we thereby come to understand only *part of* the word's meaning. Once we have this rudimentary understanding of a large enough set of words we are able to make new sentences to express propositions that might not have been stated before, and to explore the logical relations between propositions by guessing and testing. Through that process we come to understand more of the meaning of those words and sentences which we previously understood less fully. This is not any kind of *holism* about meaning or understanding: it is absurd to maintain that, to understand anything, one must understand everything. It is rather *incompletism*: we never *fully* understand anything; whatever we *do* understand, we can always understand *better*.

9. Conclusion

Existing problem-situations in World 3 prompt future discoveries and make some types of discovery more probable than others. Discovery is a creative matter. Major progress in

science involves new theories which contradict existing theories, so cannot be validly inferred from them. Even valid inference is a creative process of more or less imaginative conjecture and testing. The claim that the conclusion of a valid argument is contained in its premises is false because there are many valid arguments such that no amount or intensity of inspection or decomposition of the premises could, by itself, reveal the conclusion. Logical analysis is a matter of conjecture and testing. Inference and analysis discover the relations between propositions and thus concepts; and in making such discoveries we come to understand better the concepts and propositions that we previously understood less well. We never fully understand what we are talking about; but we can always come to understand more.

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